

# MATH 2050B Tutorial 1

Sept 14, 2016

**Exercise 1.** If  $a \in \mathbb{R}$ , prove the following:

(a)  $-(-a) = a$

(b)  $(-1)(-1) = 1$

**Exercise 2.** Let  $S = [a, b)$ , where  $a < b$ . Determine the supremum and infimum of  $S$ . Justify your answer.

**Exercise 3.** Let  $S = \{\frac{n}{2^n} : n \in \mathbb{N}\}$ . Show that  $\sup S = \frac{1}{2}$ . Also think about what  $\inf S$  is. (Hint: Use the binomial theorem to obtain an estimate of  $\frac{n}{2^n}$ ).

**Exercise 4.** Let  $A$  and  $B$  be bounded nonempty subset of  $\mathbb{R}$ , and let  $A + B := \{a + b : a \in A, b \in B\}$ . Prove that

$$\sup(A + B) = \sup(A) + \sup(B) \quad \text{and} \quad \inf(A + B) = \inf(A) + \inf(B)$$

**Exercise 5.** Let  $X$  and  $Y$  be non-empty sets and let  $h : X \times Y \rightarrow \mathbb{R}$  have bounded range in  $\mathbb{R}$ . Let  $f : X \rightarrow \mathbb{R}$  and  $g : Y \rightarrow \mathbb{R}$  be defined by

$$f(x) = \sup\{h(x, y) : y \in Y\} \quad g(y) = \inf\{h(x, y) : x \in X\}$$

Prove that

$$\sup\{g(y) : y \in Y\} \leq \inf\{f(x) : x \in X\}$$